Lattice QCD calculation of the B to Kll decay form factors

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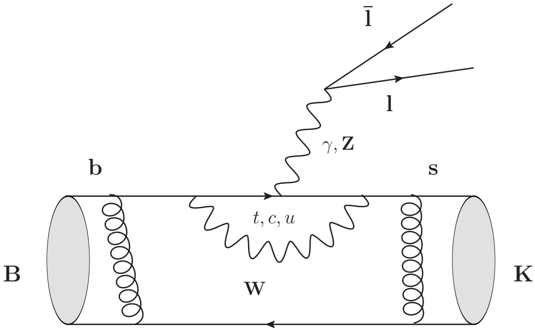
Outline

- General introduction on Lattice QCD calculations and form factors in $B \to Kll$ process.
- Detailed lattice calculations and results.
- Other on-going flavor physics projects in FNAL/MILC collaborations.
- Summary

Motivations and theoretical background

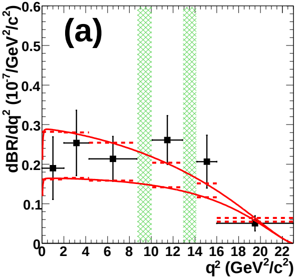
$B \rightarrow Kll$ semileptonic decay occurs through Penguin diagram ($b \rightarrow sll$).



- Standard Model (SM) contributes via FCNC (suppressed)
- Suitable process to detect physics BSM
- Studied by many experiment groups (BABAR, Belle, CDF, LHCb etc.)

Example of the observable in $B \to Kll$ process

$$\text{Theoretical predictions} = \text{Known Const.} \times f(V_{nm}) \times \langle K|\hat{O}|B\rangle$$



- $B^+ \to K^+\mu^+\mu^-$ differential branching ratio from CDF 2011
- Uncertainties in form factors are crucial to theoretical predictions(Red lines).
- High intensity front experiment(LHCb, SuperB) will come to with more accurate result.

$B \to Kll$ vs. $B \to K^*ll$ process in lattice QCD

- $B \to K^*ll$ is similar and also interesting in theory and experiment.
- K^* is unstable in nature. ($K^* \to K\pi$)
- Lattice simulations are in the unphysical parameter regime that K^* is stable.
- No ChPT theory available for $B \to K^*$.
- Extrapolation to physical regime is hard.

Form factors in $B \to Kll$ semileptonic decays

- Two matrix elements are needed in $B \to Kll$ work:

$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle$, $\langle B(p)|\bar{s}\sigma^{\mu\nu} b|K(k)\rangle$

$$\begin{aligned}\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle &= f_+\left(p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu\right) + f_0 \frac{m_B^2 - m_K^2}{q^2} q^\mu \\ &= \sqrt{2m_B}\left[f_\parallel \frac{p^\mu}{m_B} + f_\perp p_\perp^\mu\right]\end{aligned}$$

$$\begin{cases} f_\parallel(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^0 s|K(k)\rangle}{\sqrt{2m_B}} \\ f_\perp(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^i s|K(k)\rangle}{2\sqrt{m_B}} \dfrac{1}{p_i} \end{cases}$$

$$\begin{cases} f_0(E_K) = \dfrac{2m_B}{m_B^2 - m_K^2}\left[(m_B - E_K) f_\parallel(E_K) + (E_K^2 - m_K^2) f_\perp(E_K)\right] \\ f_+(E_K) = \dfrac{1}{\sqrt{2m_B}}\left[f_\parallel(E_K) + (m_B - E_K) f_\perp(E_K)\right] \end{cases}$$

Form factors in $B \to Kll$ semileptonic decays

Semileptonic $B \to K$ transition from tensor current:

$$q_\nu \langle K(k)|\bar{s}\sigma^{\mu\nu} b|B(p)\rangle = \frac{i f_T}{m_B + m_K}\left[q^2(p^\mu + k^\mu) - (m_B^2 - m_K^2)q^\mu\right]$$

Solve for f_T:

$$f_T = \frac{m_B + m_K}{\sqrt{2m_B}} \frac{\langle K(k)|ib\sigma^{0i}s|B(p)\rangle}{\sqrt{2m_B}k^i}$$

Introduction to lattice QCD

Any matrix element (observable) is given by:

$$\langle \hat{O} \rangle = \frac{1}{Z(\beta)} \int \prod_{x,\mu} dU_\mu(x) \hat{O} (\det M_F)^\delta \exp\{-S_G\}$$

We generate gauge field U with probability distribution:

$$P_U = \frac{1}{Z(\beta)} [\det M_F(U)]^\delta \exp\{-S_G(U)\} = \frac{1}{Z} \exp\{-S_{\rm eff}(U)\}$$

$$S_{\rm eff} = S_G(U) + \delta \mathrm{Tr} \ln M_F(U)$$

- $M_F(U)$ represents sea quark effect.
- Quenched n_f=0 simulation.
- Dynamical simulations n_f=2, 2+1, 2+1+1

Studies of $B \to Kll$ form factors from lattice QCD

Quenched lattice QCD:

- A. Al-Haydari et al. (QCDSF) Eur. Phys. J. A 43, 107120 (2010)
- D. Becirevic et al. Nucl. Phys. B 769, 31 (2007)
- L. Del Debbio et al. Phys. Lett. B 416, 392 (1998)
- A. Abada et al. Phys. Lett. B 365, 275 (1996)

Recent studies on dynamical Nf=2+1 flavors ensembles:

- FNAL/MILC.($B \to Kll$) hep-lat/1111.0981, hep-lat/1312.3197
- HPQCD.($B \to Kll$) Phys. Rev. D 88, 054509 (2013)
- Cambridge group.($B \to K/K^*ll$) hep-ph/1101.2726, hep-lat/1310.3722

Current lattice studies of $B \to K/K^* ll$ form factors

	FNAL/MILC	HPQCD	Cam./W/Edinb.
	$B \to K$	$B \to K$	$B \to K/K^*$
sea quark	2+1f Asqtad	2+1f Asqtad	2+1f Asqtad
valance s	Asqtad	HISQ	Asqtad
valance b	Fermilab b	NRQCD	NRQCD
used ens.	4c+5f+2sf+1uf	3c+2f	2c+1f
analysis	SχPT+z-exp.	modified z-exp.	modified z-exp.

- Three lattice groups work on the same form factors with different methods: good for the consistency check.
- The form factors will be on the whole q^2 range and can be compared with LCSR.

Lattice ensembles used in $B \to Kll$ work

$a^{-1}(fm)$	size	am_l/am_s	N_{meas}
0.12	$20^3 \times 64$	0.02/0.05	2052
0.12	$20^3 \times 64$	0.01/0.05	2259
0.12	$20^3 \times 64$	0.007/0.05	2110
0.12	$20^3 \times 64$	0.005/0.05	2099
0.09	$28^3 \times 96$	0.0124/0.031	1996
0.09	$28^3 \times 96$	0.0062/0.031	1931
0.09	$32^3 \times 96$	0.00465/0.031	984
0.09	$40^3 \times 96$	0.0031/0.031	1015
0.09	$64^3 \times 96$	0.00155/0.031	791
0.06	$48^3 \times 144$	0.0036/0.018	673
0.06	$64^3 \times 144$	0.0018/0.018	827

Table : Ensembles of QCD gauge field configurations used in the current B2K analysis. Four sources($0, \frac{N_t}{4}, \frac{N_t}{2}, \frac{3N_t}{4}$) are used for all measurements

Hadron Masses on the Lattice

For ps meson, we choose $O = \bar{q}\gamma_5 q$ as the interpretor operator:

$$C(t) =< O(t)O^{+}(0) >$$

For zero momentum projection at large t: $(O(t) = e^{Ht}O(0)e^{-Ht})$

$$\begin{aligned} C(t) &= \sum_n \frac{\langle 0|O(t)|\psi_n\rangle\langle\psi_n|O^{+}(0)|0\rangle}{2E_n} \\ &= \sum_n \frac{\langle 0|O(0)|\psi_n\rangle\langle\psi_n|O^{+}(0)|0\rangle}{2E_n} e^{-E_n t} \\ &\approx \frac{\langle 0|O(0)|\psi_0\rangle\langle\psi_n|O^{+}(0)|0\rangle}{2E_0} e^{-E_0 t} \end{aligned}$$

For ps meson correlation function measured on the lattice with periodical time boundary condition,

$$C_2(t) = A\left[e^{-mt} + e^{-m(N_t - t)}\right]$$

Hadron Masses on the Lattice

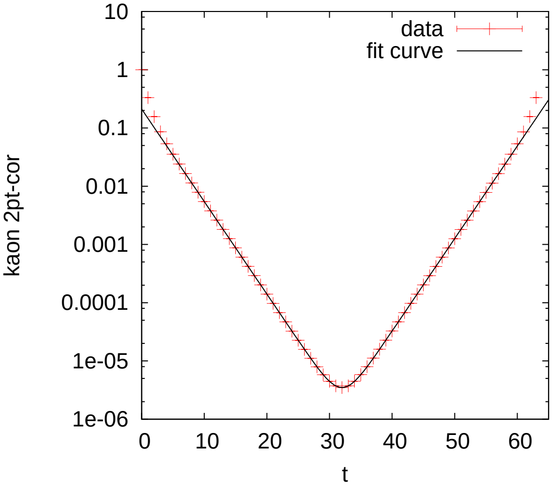


Figure : example of the kaon two-pt correlation function fit on the coarse ensemble with $am_l/am_s = 0.005/0.05$. In this test, only data from t=10-30 is used.

Hadron Masses on the Lattice

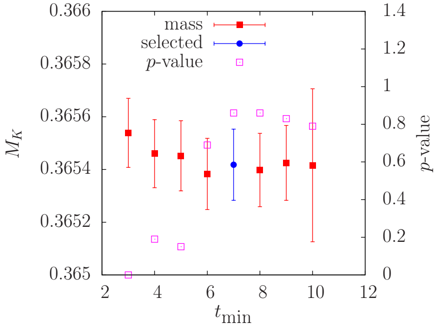


Figure : kaon (lower) mass vs. t_{min} on the coarse ensemble with $am_l/am_s = 0.005/0.05$. We fit C_2 from t_{min} to t_{max}=30.

Hadron Masses on the Lattice

For Staggered fermions, negative parity states can contribute to C_2 too, especially in B-meson case. So we fit:

$$C_2(t) = A_0\left[e^{-m_0 t} + e^{-m_0(N_t - t)}\right] + (-1)^{m(t+1)} A_1\left[e^{-m_1 t} + e^{-m_1(N_t - t)}\right] + \ldots$$

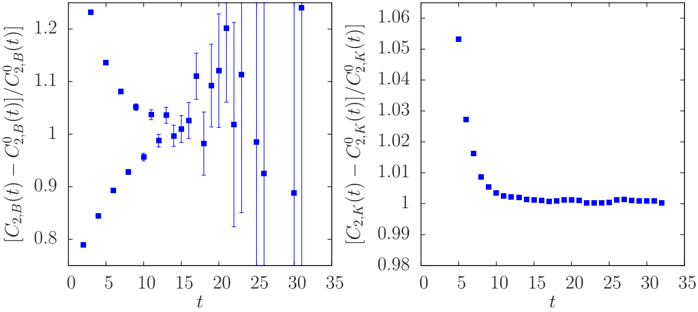


Figure : Scaled correlator $[C_2(t) - C_2^{(0)}(t)]/C_2^{(0)}(t)$ as a function of time t on the $am_l/am_s = 0.005/0.05$ coarse ensemble at the full QCD point. The oscillating opposite-parity contribution is clearly visible in the B meson correlator (left), but it is small in the zero-momentum kaon correlator (right).

Matrix elements on the lattice

We calculate matrix element from three-point correlation function:

$$C_3^\mu(t, T; \mathbf{k}) = \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{k}\cdot\mathbf{y}} \langle \mathcal{O}_K(t_{\rm src}, \mathbf{0})\, V^\mu(t_{\rm src}+t, \mathbf{y})\, \mathcal{O}_B^\dagger(t_{\rm src}+T, \mathbf{x})\rangle,$$

$$C_3^\mu(t, T; \mathbf{k}) = \sum_{m,n} (-1)^{m(t+1)}(-1)^{n(T-t-1)} A_{mn}^\mu e^{-E_K^{(m)} t} e^{-M_B^{(n)}(T-t)},$$

where

$$A_{mn}^\mu = \frac{\langle 0|\mathcal{O}_K|K^{(m)}\rangle}{2E_K^{(m)}} \langle K^{(m)}|V^\mu|B^{(n)}\rangle \frac{\langle B^{(n)}|\mathcal{O}_B|0\rangle}{2M_B^{(n)}}.$$

Matrix elements on the lattice

We use iterative averaging method to calculate matrix elements. (PRD. **79,** 054507 (2009))

$$\begin{aligned}\bar{C}_2(t) &\equiv \frac{e^{-m_P^{(0)}t}}{4}\left[\frac{C_2(t)}{e^{-m_P^{(0)}t}} + \frac{2C_2(t+1)}{e^{-m_P^{(0)}(t+1)}} + \frac{C_2(t+2)}{e^{-m_P^{(0)}(t+2)}}\right] \\ &= \frac{Z_P^2}{2m_P^{(0)}} e^{-m_P^{(0)}t} + \mathcal{O}(\Delta m_P^2),\end{aligned}$$

The negative parity states contributions are suppressed in this method. Similar trick can be applied to three-point correlation functions.

$$\bar{C}_3^{\mu(\nu)}(t, T; \mathbf{k}) \equiv A_{00} e^{-E_K^{(0)}t} \, e^{-M_B^{(0)}(T-t)} + \ldots$$

We then form the ratio:

$$\bar{R}^{\mu(\nu)}(t, T; \mathbf{k}) \equiv \frac{\bar{C}_3^{\mu(\nu)}(t, T; \mathbf{k})}{\sqrt{\bar{C}_2^K(t; \mathbf{k})\bar{C}_2^B(T-t)}} \sqrt{\frac{2E_K}{e^{-E_K^{(0)}t} \, e^{-M_B^{(0)}(T-t)}}},$$

Matrix elements on the lattice

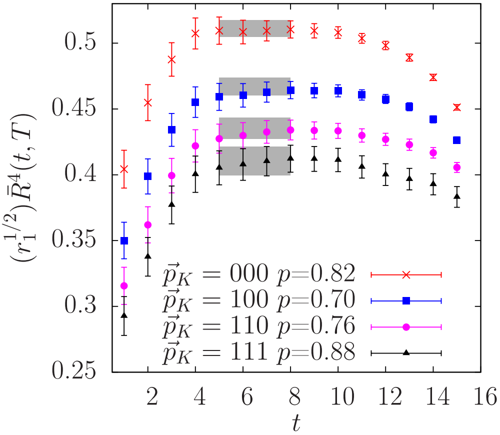


Figure : Form-factor ratio $\bar{R}^{\mu(\nu)}$ fits on the coarse ensemble with $am_l/am_s = 0.005/0.05$. The four sets of data from top to bottom correspond to lattice kaon momenta $\mathbf{k} = 2\pi(0,0,0)/L$, $2\pi(1,0,0)/L$, $2\pi(1,1,0)/L$ and $2\pi(1,1,1)/L$. We fit $\bar{R}^{\mu(\nu)}(t,T)$ to a constant, and the gray horizontal bands show the fit results with statistical errors.

Chiral-continuum extrapolation

Our form factors calculated from lattice ensembles are under these conditions:

- Unphysical heavy light quark masses. ($\frac{m_{ud}}{m_s} \sim \frac{1}{28}$ in nature)
- Non-zero lattice spacings.
- Lattice artifacts which depends on the form of the discretization.

In summary: Lattice form factors(unphysical quark masses and non-zero lattice spacings) $\rightarrow$ form factors in the continuum with physical quark masses

- Heavy-meson Staggered chiral perturbation theory is used as an effective thoery of QCD to guide the extrapolation.

Chiral-continuum extrapolation

$B_x \to P_{xy} ll$ semileptonic decays in NLO SChPT (PRD 76, 014002 (2007))

$$f = \frac{C_0}{f}(1 + \mathrm{logs} + C_1 m_x + C_2 m_y + C_3 E + C_4 E^2 + C_4 a^2)$$

$$f = \frac{C_0}{f}\left[\frac{g}{E + \Delta_B^* + D}\right] + \frac{(C_0/f)g}{E + \Delta_B^*}(\mathrm{logs} + C_1 m_x + C_2 m_y + C_3 E + C_4 E^2 + C_5 a^2)$$

where $\Delta_B^* = m_{B_s^*} - m_B$, D and logs are chiral log terms.

- For f_+ and f_T, we use vector B_s^* pole. For f_0, our test fit used a scaler B_s^* pole.
- We use SU(2), hard kaon chiral logs in the chiral fit.

Chiral-continuum extrapolation

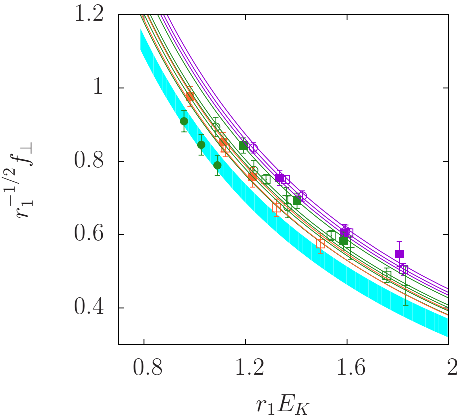


Figure : $f_\perp$ from chiral-continuum extrapolations with NLO SU(2) HMSχPT. We use bars, unfilled squares, filled squares, unfilled circles, and filled circles to denote $m_l^{\rm sea}/m_s^{\rm sea} = 0.4$, 0.2, 0.14, 0.1, and 0.05 data. We use violet, green, and orange lines to represent the fit curves on coarse, fine, and superfine ensembles. We use a cyan band to represent the continuum extrapolated curve and its error. Fit lines should pass through the data

Systematic error budget

Our continuum form factors have systematic errors from:

- Chiral-continuum extrapolation
- Heavy-light current renormalization
- Scale uncertainty
- Light- and strange-quark mass uncertainties
- Finite-volume effects
- b-quark mass correction

We study the systematic error as a function of
$q^2 = (p_B - p_K)^2 = m_B^2 + m_K^2 - 2m_B E_K$ in the lattice data E_K range.

Systematic error budget

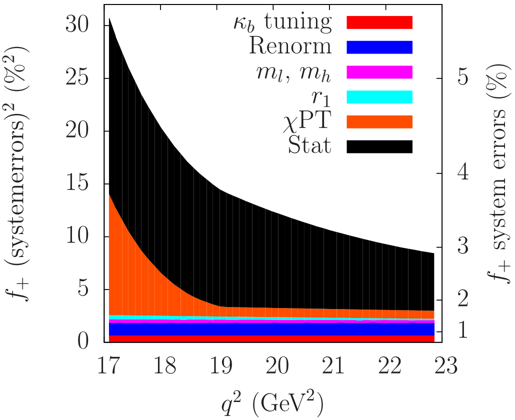


Figure : Statistical and systematic error contributions to f_+. The left y-axis label shows the squares of the errors added in quadrature, while the right y-axis label shows the errors themselves. The curves from bottom to top show the total error when we add each individual source of error one-by-one.

Systematic error budget

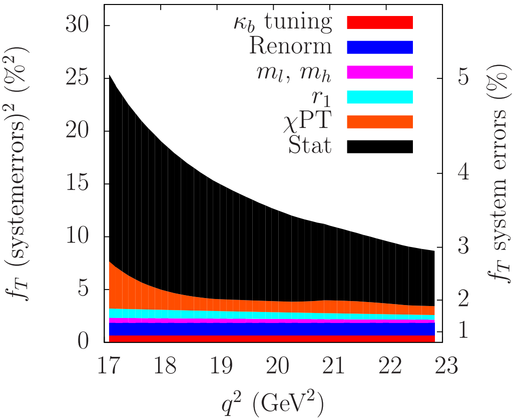


Figure : Statistical and systematic error contributions to f_T. The left y-axis label shows the squares of the errors added in quadrature, while the right y-axis label shows the errors themselves. The curves from bottom to top show the total error when we add each individual source of error one-by-one.

Systematic error budget

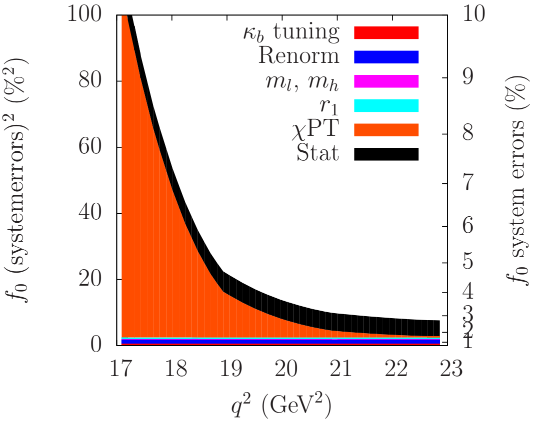


Figure : Statistical and systematic error contributions to f_0. ChPT fit error is important.

z-expansion on $B \to Kll$ form factors

What we have done:

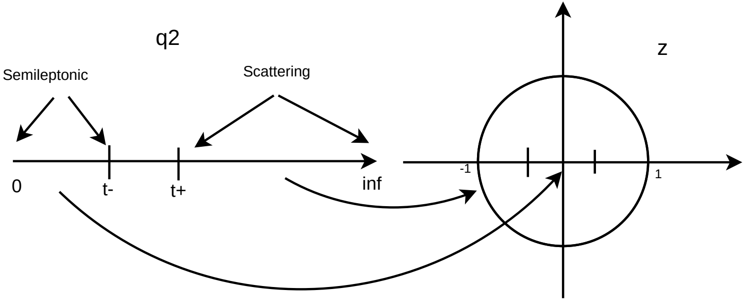
- We calculated form factors on the lattice ensembles with unphysical quark masses and non-zero lattice spacings.
- We have extrapolated the form factors back to the physical parameter regime by using SChPT.
- Form factors are functions of E_k or $q^2 = (p_B - p_K)^2 = m_B^2 + m_K^2 - 2m_B E_K$
- Our continuum form factors are in the range of $17{\sim}23.8\mathrm{GeV}^2$.

The from factors from chiral-continuum extrapolations are valid only in low E_k regime, because

- Form factors computed on the lattice are mostly in low E_k regime. (Data range is limited.)
- ChPT is valid only in low E_k regime. (Extrapolation range is limited.)

We need z-expansion as a model independent extrapolation method to get form factors in low q^2 range.

z-expansion on $B \to Kll$ form factors



- z-expansion maps q^2 to z by:

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad t_\pm = (m_B \pm m_K)^2$$

- Choose $t_0 = t_+ \left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right)$ such that $z << 1$
- Expand form factors as a function of z.

z-expansion on $B \to Kll$ form factors

- z-expansion in BGL formalism(PRL 95, 071802 (2005)):

$$f_+^{\mathrm{BGL}}(q^2) = \frac{1}{B(z)\phi(z)} \sum_{k=0}^{\infty} a_k z^k,$$

where $B(z) = z(q^2, m_R^2)$ is used to count pole structure and $\phi(z)$ is selected such that $\sum_{k=0}^{\infty} a_k^2 \leq 1$.

- A more strict constraint on $\sum_k a_k^2$ studied by Becher and Hill (PLB633:61-69 (2006)) showed

$$\sum_k a_k^2 = \frac{1}{2\pi i} \oint \frac{dz}{z} |\phi(z)B(z)f_+|^2 = \frac{m_b^2}{3} \int_{t_+}^{\infty} \frac{dt}{t^5} \left[(t - t_+)(t - t_-)\right]^{\frac{3}{2}} |f_+($$

where t is q^2. For $B \to K$, $\sum_k a_k^2 < 0.05$

z-expansion on $B \to Kll$ form factors

We use the BCL formalism of the z-expansion(PRD 79, 013008 (2009)).

$$f_+^{\mathrm{BCL}}(q^2) = \frac{1}{P(q^2)} \sum_{k=0}^{K-1} b_k \left[z^k - (-1)^{k-K} \frac{k}{K} z^K \right],$$

where $P = 1 - q^2/M_{B_s^*}^2$ is used to account the pole structure.

- The extra term $(-1)^{k-K} \frac{k}{K} z^K$ is used to reproduce the behavior of form factor near $q^2 = t_+$.
- Use $f_+^{\mathrm{BGL}} = f_+^{\mathrm{BCL}}$ to determine a matrix B such that $\sum_{m,n=0}^{K} B_{mn} b_m b_n = \sum_k a_k^2$. The bound on the coefficients is still exist.

z-expansion on $B \to Kll$ form factors

The range of q^2 and z:

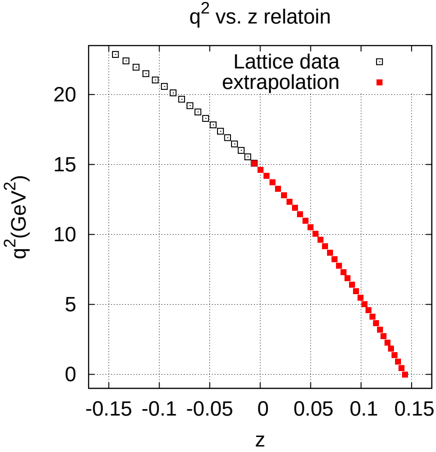


Figure : The relation between q^2 and z. The lattice data covers the range from 16 to $23.8\mathrm{GeV}^2$.

z-expansion on $B \to Kll$ form factors

Fit Pf as a function of z:

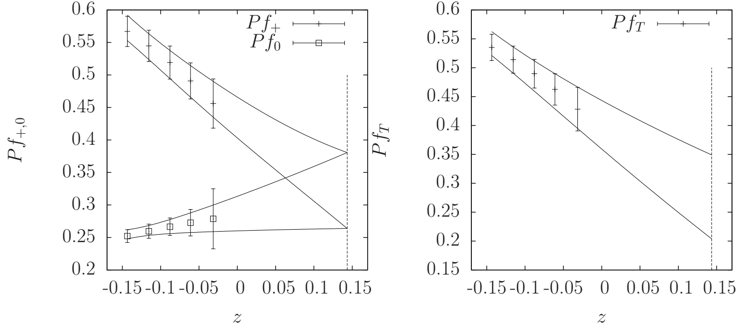
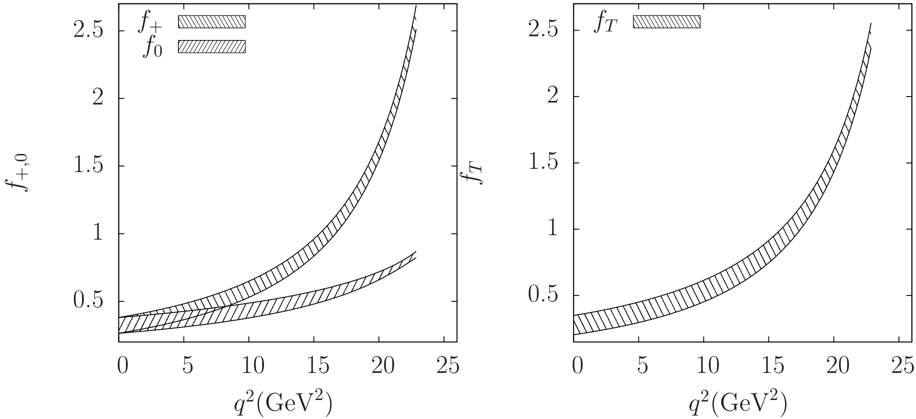


Figure : f_+, f_0, and f_T z-expansion fit plot. The synthetic data points are generated at large q^2 (small z) from LECs of HMSχPT fit result. The kinematic constraint $f_+(q^2=0) = f_0(q^2=0)$ is applied in the combined f_+ and f_0 z-expansion fit. The vertical dashed lines correspond q^2=0.

z-expansion on $B \to Kll$ form factors

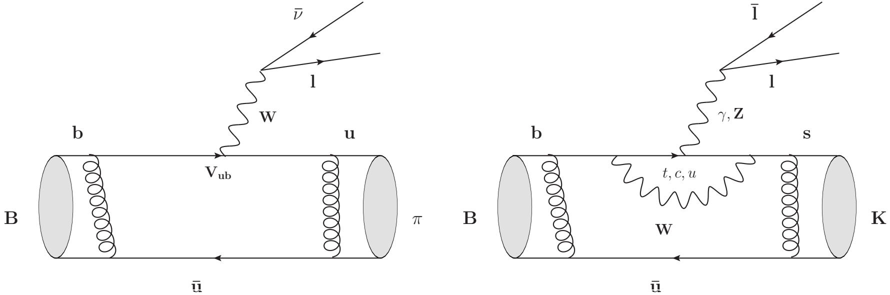
z-expansion on $B \to Kll$ form factors. (Statistical error only.)



kinematic constraint, $f_+(q^2 = 0) = f_0(q^2 = 0)$, is applied in z-expansion fit. The error on high q^2 range is decreased to 5%. (Old quenched results had 15% error.)

Other flavor physics projects in FNAL/MILC collaborations

$B \to \pi l \bar{\nu}$ semileptonic decay occurs through tree diagram ($b \to u l \bar{\nu}$).



$$
\begin{aligned}
\frac{d\Gamma}{dq^2} &\propto |V_{ub}|^2 |f_+(q^2)|^2 \quad \mathrm{Exp.} \\
\langle \pi | V^\mu | B \rangle &= f_+(q^2) \left[p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M^2 - m^2}{q^2} q^\mu \\
q^2 &= (p_B - p_\pi)^2 = M_B^2 + M_\pi^2 - 2 M_B M_\pi E_\pi
\end{aligned}
$$

$D \to \pi/Kl\nu$ semileptonic decay and $|V_{cd}|$, $|V_{cs}|$

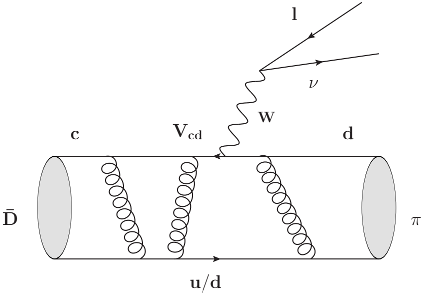


Figure : $\bar{D} \to \pi l \nu$

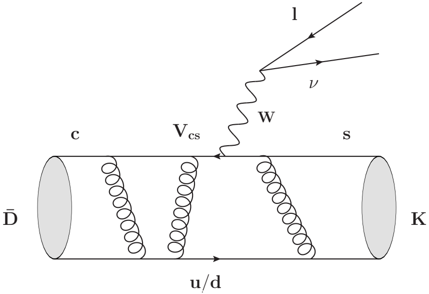


Figure : $\bar{D} \to K l \nu$

$$\frac{d\Gamma(D \to \pi/Kl\nu)}{dq^2} \propto |V_{cd/cs}|^2 |f_+|^2$$

$$\langle \pi/K|V^\mu|D\rangle = f_+(q^2)\left[p_D^\mu + p_{\pi/K}^\mu - \frac{M^2 - m^2}{q^2} q^\mu\right] + f_0(q^2)\frac{M^2 - m^2}{q^2} q^\mu$$

Summary

- Introduction form factors in lattice QCD calculation.
- Preliminary results on $B \to Kll$ form factors.
- More studies in the B, D semileptonic decay form factors from lattice calculations will be available in the future.